

Developments In 5D Quark-Lepton Symmetric Models*

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We outline some recent developments in higher dimensional Quark-Lepton (QL) symmetric models. The QL symmetric model in five dimensions is discussed, with particular emphasis on the use of split fermions. An interesting fermionic geography which utilises the QL symmetry to suppress the proton decay rate and to motivate the flavor differences in the quark and leptonic sectors is considered. The 5D quartification model is outlined and contrasted with 4D constructs.

PACS numbers: 11.10.Kk, 11.15.Ex, 14.60.Pq, 14.60.St, 14.70.Pw

The Standard Model (SM) of particle physics displays a clear asymmetry between quarks and leptons. Quarks and leptons have different masses and charges and importantly quarks experience the strong force. Despite these differences there exists a suggestive similarity between the family structure of quarks and leptons that leads one to wonder if the SM may be a low energy approximation to a more symmetric underlying theory.

How does one go about constructing a quark-lepton symmetric model? Let us recall that the SM also displays a clear asymmetry between left and right handed fields; the left handed fields experience $SU(2)_L$ interactions whilst the right handed fields do not. One generation of SM fermions may be denoted as

$$Q, L, u_R, d_R, e_R, \quad (1)$$

revealing a further left-right asymmetry; namely the absence of ν_R . However the left-right asymmetry may be a purely low energy phenomenon and the construction of a high energy left-right symmetric theory proceeds as follows. One must first extend the SM particle content to include ν_R and thereby equate the number of left and right degrees of freedom in the model. The SM gauge group must also be enlarged [1, 2, 3, 4]:

$$SU(3) \otimes SU(2) \otimes U(1) \rightarrow SU(3) \otimes [SU(2)]^2 \otimes U(1).$$

This enables one to define a discrete symmetry interchanging all left and right handed fermions in the Lagrangian, $f_L \leftrightarrow f_R$. Furthermore the model must be constructed such that the additional symmetries introduced to enable the left-right interchange invariance are suitably broken to reproduce the SM at low energies.

One may construct a quark-lepton symmetric model by employing the same recipe [5, 6, 7, 8, 9, 10, 11]. First one must extend the fermion content of the SM. As quarks come in three

colors one is required to introduce more leptons to equate the number of quark and lepton degrees of freedom. For each SM lepton one includes two exotic leptons,

$$\begin{aligned} e &\rightarrow E = (e, e', e''), \\ \nu &\rightarrow N = (\nu, \nu', \nu''), \end{aligned} \quad (2)$$

where the primed states are the exotics (known as liptons in the literature). The gauge group must also be extended:

$$SU(3) \otimes SU(2) \otimes U(1) \rightarrow [SU(3)]^2 \otimes SU(2) \otimes U(1),$$

where the additional $SU(3)$ gauge bosons induce transitions amongst the generalised leptons in the same way that gluon exchange enables quarks to change color. Now one is able to define a discrete symmetry whereby one interchanges all quarks and (generalised) leptons in the Lagrangian,

$$Q \leftrightarrow L, \quad u_R \leftrightarrow E_R, \quad d_R \leftrightarrow N_R. \quad (3)$$

The gauge symmetry may be broken via the Higgs mechanism to reproduce the charge differences between quarks and leptons and give heavy masses to the liptons. Phenomenologically consistent models can be obtained by breaking the lepton color group $SU(3)_\ell$ completely or by leaving an unbroken subgroup $SU(2)_\ell \subset SU(3)_\ell$ (which serves to confine the liptons into exotic bound states). The QL symmetry implies mass relations of the type $m_u = m_e$ which are more difficult to rectify. In 4D one may remove these by extending the scalar sector and thereby increasing the number of independent Yukawa couplings.

In recent years the study of models with extra dimensions has revealed a number of new model building tools. The use of orbifolds provides a new means of reducing the gauge symmetry operative at low energies by introducing a new mass scale, namely the compactification scale $M_c = 1/R$. In generic 5D models the mass of exotic gauge bosons can be set by M_c and the reduction of symmetries by orbifold construction results in collider phenomenology which differs from that obtained in models employing the usual 4D Higgs symmetry breaking.

The reduction of the QL symmetric gauge group via orbifold construction has recently been studied in 5D [12]. One question that faces the model builder in extra dimensional models is whether to place fermions on a brane or in the bulk. We shall focus on the latter in what follows and demonstrate

*Talk given by K.L.McDonald at the Joint Meeting of Pacific Region Particle Physics Communities, Honolulu, Hawaii, October 29th - November 3rd 2006

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that the troublesome mass relations which arise in 4D QL models actually provide useful and interesting model building constraints in 5D models.

In five dimensions bulk fermions lack chirality. However, chiral zero mode fermions, which one may identify with SM fermions, may be obtained by employing orbifold boundary conditions on a fifth dimension forming an S^1/Z_2 orbifold. By coupling a bulk fermion to a bulk scalar field one may readily localise a chiral zero mode fermion at one of the orbifold fixed points [13]. Denoting the bulk fermion (scalar) as ψ (ϕ) one has the following Lagrangian:

$$\mathcal{L} = \bar{\psi}(\Gamma_M \partial^M - f\phi)\psi - \frac{1}{2}\partial_M \phi \partial^M \phi - V(\phi), \quad (4)$$

where Γ_M are the Dirac matrices, f is a constant, $M = 0, 1, \dots, 4$ is the 5D Lorentz index and

$$V(\phi) = \frac{\lambda}{4}(\phi - v)^2, \quad (5)$$

is the usual quartic potential. If ϕ transforms trivially under Z_2 its ground state is given by $\langle \phi \rangle = v$. However if ϕ is odd under Z_2 its ground state is required to vanish at the fixed points. This results in a kink vacuum profile for ϕ which serves to localise chiral zero mode fermions, ψ_0 , at one of the orbifold fixed points. The fixed point at which ψ_0 is localised depends on the sign of f .

In a QL symmetric model one must specify the transformation properties of the bulk scalar under the QL symmetry. An interesting choice is to make ϕ odd under the QL symmetry [14], resulting in a Yukawa Lagrangian of the form

$$\mathcal{L} = -\{f_Q(Q^2 - L^2) + f_u(U^2 - E^2) + f_d(D^2 - N^2)\}\phi,$$

where $Q^2 = \bar{Q}Q$, etc, and the SM fermions are identified with the chiral zero modes of the bulk fermions in an obvious fashion. Note that the choice $f_Q, f_u, f_d > 0$ automatically implies the localisation pattern shown in Figure 1. It is of interest that this geography may be implemented in a less arbitrary fashion in a QL symmetric framework as this pattern is precisely that advocated recently to suppress the proton decay rate without an extremely large ultra-violet cutoff [15]. Indeed the effective 4D proton decay inducing non-renormalizable operator has the form

$$\mathcal{O}_p = \frac{K}{\Lambda^2} \mathcal{O}_Q^3 \mathcal{O}_L \quad (6)$$

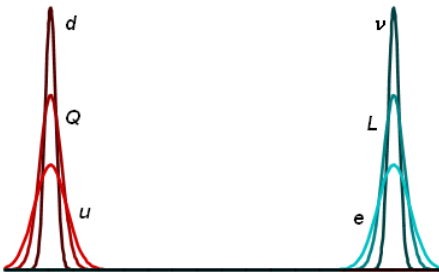


FIG. 1: The 5D wavefunctions for quarks and leptons in a QL symmetric model with one bulk scalar.

where \mathcal{O}_Q (\mathcal{O}_L) generically denotes a quark (lepton) operator and $K \sim \exp\{-vL^{3/2}\}$ represents the wavefunction overlap between quarks and leptons in the extra dimension. L denotes the length of the fundamental domain of the orbifold.

Observe that fermions related by the QL symmetry necessarily develop identical wavefunction profiles in the extra dimension and consequently the troublesome mass relations of the type $m_e = m_u$ persist in the effective 4D theory. With one bulk scalar it is only possible to localise fermions at the orbifold fixed points. However two bulk scalar models enable one to localise fermions within the bulk. This works as follows [16, 17]. With one bulk scalar chiral zero mode fermions are found at one of the orbifold fixed points, with the precise point of localisation determined by the sign of the relevant Yukawa coupling. If a second bulk scalar is added with an opposite sign Yukawa coupling it will tend to drag the fermion towards the other end of the extra dimension, thereby localising it in the bulk. Importantly though the chiral fermion cannot be pulled very far into the bulk before it is dragged all the way to the other end of the extra dimension.

Let us add a second bulk scalar which is even under the QL symmetry, giving rise to the Lagrangian

$$\mathcal{L}_2 = -\{h_Q(Q^2 + L^2) + h_u(U^2 + E^2) + h_d(D^2 + N^2)\}\phi'.$$

Let us again require $h_Q, h_u, h_d > 0$ so that all quark Yukawa couplings are positive. Both ϕ and ϕ' will attempt to localise quarks at the same point in the extra dimension and they will remain at their original point of localisation. This type of quark geography is precisely that advocated in [20] to allow one to construct quark flavor without introducing large flavor changing neutral currents. However ϕ and ϕ' attempt to localise leptons at different fixed points and the resultant point of localisation for a given lepton depends on which scalar dominates (these statements will be made numerically precise in a forthcoming publication [18]). An arrangement typical of this setup is shown in Figure 2.

This arrangement has a number of interesting features. Firstly note that shifting leptons into the bulk significantly alters the degree of wavefunction overlap in the quark and leptonic sectors. When one obtains the effective 4D fermion masses the different degree of overlap in the quark and lepton sectors will remove the undesirable QL mass relations. Furthermore the overlap between left and right chiral fermion

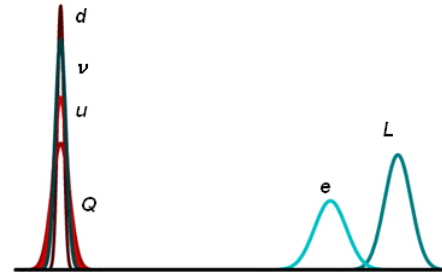


FIG. 2: The 5D wavefunctions for quarks and leptons in a QL symmetric model with two bulk scalars.

wavefunctions is expected to be greater in the quark sector than in the lepton sector, leading to the generic expectation that quarks will be heavier than the lepton to which they're related by the QL symmetry. Observe that dragging the right chiral neutrinos ν_R all the way to the 'quark end' of the extra dimension significantly suppresses the neutrino Dirac masses below the electroweak scale. Dragging ν_R to the quark end of the extra dimension does not introduce rapid proton decay, provided that ν_R has a large enough Majorana mass to kinematically preclude decays of the type $p \rightarrow \pi \nu_R$. Such a mass is expected to arise at the non-renormalizable level even if it is not induced by tree-level couplings in the theory [19].

Thus the non-desirable Yukawa relations implied by 4D QL symmetric models turn out to be of interest in the 5D construct. They enable one to understand proton longevity within the split fermion framework in a less arbitrary fashion and instead of inducing unwanted mass relations suggest an underlying motivation for the flavour differences experimentally observed in the quark and lepton sectors.

Adding a leptonic color group to the SM clearly renders the traditional approaches to gauge unification inapplicable. Recall that the simplest grand unified theory, namely $SU(5)$, does not contain the left-right symmetric model. However there are larger unifying groups which do admit the left-right symmetry, for example $SO(10)$ and the trinification gauge group $[SU(3)]^3 \times Z_3$. Similarly it is possible to construct a unified gauge theory admitting the QL symmetry by considering larger unification groups. It is natural to extend the notion of trinification to include a leptonic color factor, leading one to the so called quartification model. This possesses the gauge group $G_Q = [SU(3)]^4 \times Z_4$, where the additional $SU(3)$ factor corresponds to lepton color and the Z_4 symmetry cyclicly permutes the group factors to ensure a single coupling constant [21, 22, 23, 24, 25, 26].

It has been demonstrated that unification may be achieved within the quartification framework in 4D by enforcing additional symmetries upon the quartification model [22]. Subsequent work has shown that unification need not require additional symmetries, but does require multiple symmetry breaking scales between the unification scale and the electroweak scale [24]. It was also shown in [24] that unification may be achieved via multiple symmetry breaking routes both with and without the remnant leptonic color symmetry $SU(2)_\ell$.

The necessary symmetry breaking is accomplished with eight Higgs multiplets, giving rise to a complicated Higgs potential with a large number of free parameters. The demand of multiple symmetry breaking scales also requires hierarchies of vacuum expectation values (VEV's) to exist within individual scalar multiplets, giving rise to a generalised version of the doublet-triplet splitting problem familiar from $SU(5)$ unified theories. A large number of electroweak doublets also appear in the 4D constructs. Thus many of the less satisfactory features of 4D quartification models revolve around the Higgs sector induced symmetry breaking.

Recently the quartification model has been studied in 5D, where intrinsically higher dimensional symmetry breaking methods exist [26]. By taking the fifth dimension as an $S^1/Z_2 \times Z'_2$ orbifold one may employ orbifold boundary con-

ditions (OBC's) on the bulk gauge sector to reduce the gauge symmetry operative at the zero mode level from G_Q to

$$SU(3)_c \otimes SU(2)_L \otimes SU(2)_\ell \otimes SU(2)_R \otimes U(1)^3. \quad (7)$$

However the use of OBC's does not reduce the rank of the gauge group so that further symmetry breaking is required. Rank reducing symmetry breaking can be achieved in higher dimensional theories by employing a boundary scalar sector to alter the boundary conditions on the compactified space for gauge fields [27]. Denoting a boundary scalar as χ and defining $V \propto \langle \chi \rangle$ one can show that V induces a shift in the Kaluza-Klein mass spectrum of gauge fields which couple to χ . If such a gauge field initially possessed a massless mode its tower receives a shift of the form

$$M_n \approx M_c (2n+1) \left(1 + \frac{M_c}{\pi V} + \dots \right), \quad n = 0, 1, 2, \dots \quad (8)$$

giving a tower with the lowest-lying states $M_c, 3M_c, 5M_c, \dots$. This represents an offset of M_c relative to the $V = 0$ tower, with the field no longer retaining a massless zero mode. The association of V with the VEVs of the boundary scalar sector implies that the limit $V \rightarrow \infty$ is attained when $\langle \chi \rangle \rightarrow \infty$. However, when the VEVs of the Higgs fields are taken to infinity, the shift in the KK masses of the gauge fields is finite, giving the exotic gauge fields masses dependent only upon the compactification scale M_c . Consequently, these fields remain as ingredients in the effective theory while the boundary Higgs sector decouples entirely, and we can view our reduced symmetry theory in an effective Higgsless limit. Interestingly, in this limit also, the high-energy behaviour of the massive gauge boson scattering remains unspoilt as shown in [27].

It was shown in [26] that a unique set of OBC's was required to ensure that quark masses could be generated and to prevent liptons from appearing at the electroweak scale. The inclusion of a boundary Higgs sector allows one to reduce the quartification gauge symmetry down to the SM gauge group G_{SM} or to $G_{SM} \otimes SU(2)_\ell$ at the zero mode level. In both cases fifth dimensional components of the $SU(3)_L$ gauge fields with the quantum numbers of the SM Higgs doublet retained a massless mode, enabling one to use Wilson loops to reproduce the SM flavour structure.

A surprising result however was that unification could only be achieved when the remnant lepton color symmetry $SU(2)_\ell$ remained unbroken. Thus one arrives at a unique minimal quartification model which unifies in 5D, a result to be contrasted with the 4D case where a large number of symmetry breaking routes which permit unification have been uncovered. Unfortunately the unifying case requires the compactification scale to be greater than 10^{10} GeV so that only a SM like Higgs field is expected to appear at the LHC.

Quark-lepton symmetric models in some sense unify the fermionic content of the SM and thereby motivate the similar family structures observed in the quark and lepton sectors. Recent investigations involving QL symmetries in 5D have uncovered a number of interesting results. In particular the QL symmetry provides useful Yukawa relationships in

split fermion models and the quartification model is found to be more constrained in 5D. A number of avenues for further investigation remain, with a detailed analysis of the fermionic geography of Figure 2 required [18]. It would also be interesting to combine the QL symmetry with a left-right symmetry in 5D, enabling one to simultaneously employ results uncovered in each of these frameworks [28].

Acknowledgements

KM thanks the organisers of the Joint Meeting of Pacific Region Particle Physics Communities 2006 for a splendid conference and R. Foot and R. Volkas for helpful communications.

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